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Analytic solution to Indefinite Linear Quadratic Regulator for Stochastic Systems

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Abstract: This paper aims to deal with the indefinite linear quadratic regulator(ILQR) for stochastic systems. It provides both the analytic solution to the ILQR and the sufficient and necessary condition under which the ILQR is solvable. Different from the existed literature, we obtain the results in a novel way. In order to obtain a tighter necessary condition, we investigate the solution structure of a two-point boundary value problem for a stochastic differential equations directly but involve no complicated probabilistic derivation. Consider that there are close relationships between the linear game problem and H_∞ control and ILQR, the idea in the paper can also be extended to solve these problems.

Key Words: linear quadratic regulator, stochastic systems, forward backword stochastic differential equations, Riccati equation

1 Introduction

Linear quadratic regulator(LQR) problem is one of the most important issues in control theory. The problem in deterministic setting had been solved completely. It is well known that its most elegant solution and solvable condition are based on the solution to a Riccati equation determined by the system and performance parameters. It is worth stressing that the aforementioned elegant results are only restricted to the case that LQR has the positive definite control weighting matrix in the cost functional. Few literature is devoted to the LQR with the indefinite weighting matrix. With the rapid development of modern control theory, attention of researcher was first attracted by LQR in stochastic setting with definite weighting matrix [1], and then shifted by LQR in stochastic setting with indefinite weighting matrix [2–5]. Stochastic LQR problems have been first studied by Wonham [6]and by many researchers later (cf., e.g., [7, 8]), but the assumption that the control weighting matrix is positive definite has been taken for granted in all of these works. Until 1998, the authors of [3] observed that some stochastic LQR problems with nonzero control diffusion coefficient still make sense when the control weighting matrix is indefinite or even negative definite. The intristic cause of the phenomenon can be derived from that the gain in cost due to a larger control size may not outweigh the loss due to a greater uncertainty in systems . [3] also associated the solvability of a stochastic Riccati equation with the well-posedness and solution to ILQR but it only presented a sufficient condition based on stochastic Riccati equation. [9] was concerned with the ILQR for which the coefficients are allowed to be random. The ILQR for stochastic systems is solved completely in [2], where the analytic solution and sufficient and necessary condition are provided based on the solution to a constrained generalized Riccati equaltion. Different from the proof, in the paper, we present a novel idea to get the same results. In detail, we focus on solve the problem via deriving the relationship between state and co-state as well as the solution structure to the two-point boundary value problems for stochastic systems instead of their solutions [10, 11].

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The remainder of this paper is organized as follows. Section 2 is devoted to the general LQR for deterministic system, which will be helpful to understand the idea we solve the ILQR for stochastic systems in Section 3. Finally, Section 4 gives some conclusions.

Notation: Let $(\Omega, P, \mathcal{F}, \{\mathcal{F}_t\}_{\geq 0}, \mathcal{P})$ be a complete probability space on which a one-dimensional standard Brownian motion $W(t)$ is defined such that $\{\mathcal{F}_t\}_{t\geq 0}$ is the natural filtration generated by $W(t)$, augmented by all the $\mathcal{P}$-null sets in $\mathcal{F}$. All the stochastic systems involved in the paper are on $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{\geq 0}, \mathcal{P})$. R_x stands for Euclidean space with the same dimension with x. $\langle\cdot,\cdot\rangle$ represents inner product in Euclidean space. Let $L^2_{\mathcal{F}}(0,T;R_x)$ be the set of all $\{\mathcal{F}_t\}$-progressively measurable processes $X(\cdot)$ valued in R_x such that $\int_0^T E|X(t)|^2dt < \infty$ and $L_{\mathcal{F}_T}(0,T;R_u)$ be be the set of all $\{\mathcal{F}_T\}$- measurable processes $X(\cdot)$ valued in R_u. M^+ denotes the Moore-Penrose inverse of a matrix. $M > (\geq)0$ means that M is positive definite (semidefinite). In order to simplify the statement, we omit argument of many functions when there is no confusion. In addition, it should be noted that the paper has abuse of notations, for instance, P, appearing in both deterministic LQR and stochastic LQR, satisfies different Riccati equations in different sections.

2 LQR for deterministic systems

To make the paper more accessible and to show that the derivation is not trivial, we will start with recall a deterministic LQR.

Introduce the deterministic system

$$\dot{x}(t) \quad = \quad Ax(t) + Bu(t) \tag{2.1}$$

where x and u are state and control input, respectively, A and B are known matrices. Introduce the cost function

$$J = \int_0^T (x'Qx + u'Ru)dt + x(T)'Px(T), \tag{2.2}$$

where $Q \geq 0$ and $R > 0$ respectively.

Now the LQR problems for deterministic systems can be stated as : find the admissible controller $u(t)$ such that the cost function J in (2.1) subject to (2.1) is minimized.

For the LQR problem in deterministic setting, the corresponding Hamiltonian is

$$\mathcal{H} = x'Qx + u'Ru + p'(Ax + Bu). \tag{2.3}$$

According to maximum principle, the optimal controller for (2.2) subject to (2.1) should satisfy

$$\frac{\partial \mathcal{H}}{\partial u} = 0 \tag{2.4}$$

i.e.

$$u = -B'p \tag{2.5}$$

and p is governed by

$$-\dot{p} = \frac{\partial \mathcal{H}}{\partial x}, p(T) = P_T x(T) \tag{2.6}$$

i.e.

$$\dot{p} = -Qx - A'p, p(T) = P_T x(T) \tag{2.7}$$

Because $R > 0$, the minimal problem is convex over $u(t)$. Hence, the solvability of the problem is then equivalent to that of the two-point boundary values problem for the ordinary differential system as

$$\begin{aligned} \dot{x}(t) &= Ax - BB'p, x(0) = x_0 & (2.8)\\ \dot{p}(t) &= \dot{p} = -Qx - A'p, p(T) = P_T x(T) & (2.9) \end{aligned}$$

Lemma 2.1 *The solution (x, p) to the system (2.8)-(2.9) admits*

$$p(t) = P(t)x(t) \tag{2.10}$$

The proof will be given in the later.

We will start with considering a more general results about the solution to the forward-backward ordinary differential equations. To avoid the notation complexity, the below will omit the time index and we believe that it will not cause confusions.

Lemma 2.2 *For given $f_1(t), f_2(t)$, the solution (x, p) to the system*

$$\begin{aligned} dx &= [Ax + Bp + f_1]dt, x(0) = x_0 & (2.11)\\ dp &= [Cx + Dp + f_2]dt, p(T) = Fx(T) & (2.12) \end{aligned}$$

admits

$$p(t) = P(t)x(t) + r(t) \tag{2.13}$$

where $P(t)$ is independent of f_1, f_2, $r(t)$ admits differential relation driven by f_1, f_2.

Proof: Denote $\bar{A} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}, \bar{f} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$,

$$\begin{bmatrix} x(t) \\ p(t) \end{bmatrix} = e^{\bar{A}t} \begin{bmatrix} x(0) \\ p(0) \end{bmatrix} + \int_0^t e^{\bar{A}(T-s)} \bar{f} ds \tag{2.14}$$

Let

$$e^{\bar{A}t} = \begin{bmatrix} l_{11}(t) & l_{12}(t) \\ l_{21}(t) & l_{22}(t) \end{bmatrix} \tag{2.15}$$

$$\int_0^t e^{\bar{A}(t-s)} \bar{f} ds = \begin{bmatrix} \bar{l}_{11}(t) & \bar{l}_{12}(t) \\ \bar{l}_{21}(t) & \bar{l}_{22}(t) \end{bmatrix} \tag{2.16}$$

then the equation is equivalent to

$$\begin{aligned} x(t) &= l_{11}(t)x(0) + l_{12}(t)p(0) + \bar{l}_{11}(t) + \bar{l}_{12}(t) & (2.17)\\ p(t) &= l_{21}(t)x(0) + l_{22}(t)p(0) + \bar{l}_{21}(t) + \bar{l}_{22}(t) & (2.18) \end{aligned}$$

Combining $P(T) = Fx(T)$ with (2.17) and (2.18), one has

$$\begin{aligned} & l_{21}(T)x(0) + l_{22}(T)p(0) + \bar{l}_{21}(T) + \bar{l}_{22}(T) \\ = \; & F(l_{11}(T)x(0) + l_{12}(T)p(0) + \bar{l}_{11}(T) + \bar{l}_{12}(T)) \end{aligned} \tag{2.19}$$

The relationship (2.19) immediately shows that $p(0)$ is affine in $x(0)$. With the similar lines, as the system is initialized at any instant s, it is then not hard to get that $p(s)$ is affine in $x(s)$, i.e.

$$p(s) = P(s)x(s) + r(s). \tag{2.20}$$

In addition, if let (x_1, p_1) and (x_2, p_2) be solutions to (2.17) and (2.18) with initial value x_{1i} and x_{2i} respectively. It is not difficult to check that $(x_1, p_1) - (x_2, p_2) = (\bar{x}, \bar{p})$ i.e.

$$\begin{aligned} \bar{x}(t) &= l_{11}(t)\bar{x}(0) + l_{12}(t)\bar{p}(0) & (2.21)\\ \bar{p}(t) &= l_{21}(t)\bar{x}(0) + l_{22}(t)\bar{p}(0) & (2.22) \end{aligned}$$

is actually the solution to the following homogeneous system

$$\begin{aligned} d\bar{x} &= [A\bar{x} + B\bar{p}]dt, \bar{x}(0) = \bar{x}_0 & (2.23)\\ d\bar{p} &= [C\bar{x} + D\bar{p}]dt, \bar{p}(T) = F\bar{x}(T) & (2.24) \end{aligned}$$

where initial value $\bar{x}_0 = x_{1i} - x_{2i}$. From (2.21), (2.22) and $\bar{p}(T) = F\bar{x}(T)$, one will derive that

$$\bar{p}(0) = P(0)\bar{x}(0) \tag{2.25}$$

If the system (2.23)-(2.24) is initialized at any instant s instead of at 0 instant, it is then not hard to get that $\bar{p}(s)$ is linear in $\bar{x}(s)$, i.e.

$$\bar{p}(s) = P(s)\bar{x}(s). \tag{2.26}$$

In order to characterize $P(t)$, we only need to associate with (2.23)-(2.24) to differential $\bar{p} = P\bar{x}$ and then compare the two-side coefficients of the equation, which also means that P is indeed independent of f_1, f_2. The proof is now complete.

Lemma 2.2 shows that the solution to the inhomogeneous two-point boundary values problem for ODEs has special structure as (2.13).

Theorem 2.1 *If the LQR problem is solvable, P in (2.10) admits*

$$0 = \dot{P} + PA' + AP - PBR^{-1}B'P + Q, P(T) = P_T \tag{2.27}$$

and the optimal controller is given as

$$u(t) = -R^{-1}B'Px(t) \tag{2.28}$$

Proof From Lemma 2.2, we can substitute (2.10) into (2.9) and get

$$(\dot{P} + PA' + AP - PBB'P + Q)x = 0 \tag{2.29}$$

If the initial value of the system (2.8) is given arbitrarily, so is the trajectory of $x(t)$. Now the results (2.27) is direct.

Theorem 2.2 *The LQR problem is solvable if and only if Riccati equation (2.27) has positive semi-definite solution.*

Proof With the priori discussion, there only needs to prove that P in (2.27) is positive semi-definite. From (2.9), we can get the optimal J satisfies

$$J = \langle x(0), p(0)\rangle = \langle x(0), Px(0)\rangle \tag{2.30}$$

It is clear that $J \geq 0$. Given that $x(0)$ is arbitrary, P is necessarily positive semi-definite.

3 LQR for stochastic systems

Consider the stochastic system

$$\begin{aligned} dx(t) &= [Ax(t) + Bu(t)]\,dt \\ &\quad + [A_0x(t) + B_0u(t)]\,dW(t) \end{aligned} \tag{3.31}$$

where x and u are state and control input, respectively, and A, B, A_0 and B_0 are known matrices,and cost function

$$J = E[\int_0^T (x'Qx + u'Ru)dt + x(T)'P_Tx(T)] \tag{3.32}$$

where E stands for taking mathematical expectation, R, Q and P_T are given symmetric matrices.

Now the LQR problems for stochastic systems can be stated as : find the admissible controller $u(t)$ such that the cost function J in (3.32) subject to (3.31) is minimized.

Its corresponding Hamiltonian is

$$\begin{aligned} \mathcal{H} &= x'Qx + u'Ru + p'(Ax + Bu) \\ &\quad + H'(A_0x + B_0u) \end{aligned} \tag{3.33}$$

The optimal controller satisfies:

$$\frac{\partial \mathcal{H}}{\partial u} = 0 \tag{3.34}$$

i.e.

$$Ru = -B'p - B_0'H \tag{3.35}$$

where (p, H) satisfies the adjoint equation

$$-dp = \frac{\partial \mathcal{H}}{\partial x}dt - HdW, p(T) = P_Tx(T) \tag{3.36}$$

i.e.

$$\begin{aligned} dp &= (-Qx - A'p - A_0'H)dt + HdW, \\ &\quad p(T) = P_Tx(T) \end{aligned} \tag{3.37}$$

The solvability of the problem is then equivalent to that of a two-points boundary values problem for the forward-backward stochastic differential equations.

$$\left\{ \begin{array}{rcl} dx(t) &=& [Ax(t) + Bu(t)]\,dt \\ && + [A_0x(t) + B_0u(t)]\,dW, x(0) = x_0 \\ dp(t) &=& -[Qx(t) + A'p(t) + A_0'H]dt \\ && + HdW, p(T) = P_Tx(T) \end{array} \right. \tag{3.38}$$

with

$$Ru = -B'p - B_0'H \tag{3.39}$$

Lemma 3.1 *If the LQR problem for the stochastic system (3.31) is solvable, then the solution (x, p) to the system (3.38)-(3.39) admits*

$$p(t) = P(t)x(t). \tag{3.40}$$

Along the similar lines with the deterministic case but with nontrivial generalization, we will start with investigating the solution to a more general forward-backward stochastic system:

$$\begin{aligned} dx &= [Ax + Bp + \bar{B}H + f_1]dt \\ &\quad + [A_0x + B_0p + \bar{B}_0H + g_1]dW, x(0) = x_0 \end{aligned} \tag{3.41}$$

$$\begin{aligned} dp &= [Cx + Dp + \bar{D}H + f_2]dt \\ &\quad + [C_0x + D_0p + \bar{D}_0H + g_2]dW, P(T) = Fx(T) \end{aligned} \tag{3.42}$$

Lemma 3.2 *Assume the forward backward stochastic differential equations (3.41)-(3.42) have a solution, then the solution (x, p) admits*

$$p(t) = L(t)x(t) + l(t) \tag{3.43}$$

where $L(t)$ is independent of the inhomogeneous term in the above system and decided by

$$\begin{aligned} dx &= [Ax + Bp + \bar{B}H]dt \\ &\quad + [A_0x + B_0p + \bar{B}_0H]dW, x(0) = x_0 \end{aligned} \tag{3.44}$$

$$\begin{aligned} dp &= [Cx + Dp + \bar{D}H]dt \\ &\quad + [C_0x + D_0p + \bar{D}_0H]dW, P(T) = Fx(T) \end{aligned} \tag{3.45}$$

and $l(t)$ admits a differential relation driven by f_1, f_2, g_1, g_2.

Proof To analyze the solution structure of the system (3.44)-(3.45) , we introduce the system

$$\begin{aligned} dX(t) &= [\mathcal{A}X(t) + \mathcal{B}H + \bar{f}(t)]dt \\ &\quad + [\mathcal{A}_0X(t) + \mathcal{B}_0H + \bar{g}(t)]dW \end{aligned} \tag{3.46}$$

where

$$X(t) = \left[\begin{array}{c} x(t) \\ p(t) \end{array} \right], \tag{3.47}$$

$$\mathcal{A} = \left[\begin{array}{cc} A & B \\ C & D \end{array} \right], \mathcal{A}_0 = \left[\begin{array}{cc} A_0 & B_0 \\ C_0 & D_0 \end{array} \right], \tag{3.48}$$

$$\mathcal{B} = \left[\begin{array}{c} \bar{B} \\ \bar{D} \end{array} \right], \mathcal{B}_0 = \left[\begin{array}{c} \bar{B}_0 \\ \bar{D}_0 \end{array} \right] \tag{3.49}$$

$$\bar{f}(t) = \left[\begin{array}{c} f_1 \\ f_2 \end{array} \right], \bar{g}(t) = \left[\begin{array}{c} g_1 \\ g_2 \end{array} \right]. \tag{3.50}$$

According to the variation of constant formula, the solution to the system (3.46) can be read as

$$\begin{aligned} X(t) &= \phi(t)X(0)+\phi(t)\int_0^t \phi(s)^{-1}(\mathcal{B}-\mathcal{A}_0\mathcal{B}_0)Hds \\ &\quad +\int_0^T \phi(s)^{-1}[\bar{f}-\mathcal{A}_0\bar{g}]ds \\ &\quad +\int_0^T \phi(s)^{-1}\mathcal{B}_0HdW+\int_0^T \phi(s)^{-1}\bar{g}dW \end{aligned} \tag{3.51}$$

where

$$d\phi=\mathcal{A}\phi dt+\mathcal{A}_0\phi dW \tag{3.52}$$

By virtue of $P(T)=Fx(T)$, now it is direct to get the relationship i.e.

$$\begin{aligned} &\left[\begin{array}{cc} F & -I \end{array}\right]\phi(T)\left\{X(0)+\int_0^T \phi(s)^{-1}(\mathcal{B}-\mathcal{A}_0\mathcal{B}_0)Hds\right. \\ &+\int_0^T \phi(s)^{-1}[\bar{f}-\mathcal{A}_0\bar{g}]ds \\ &\left.+\int_0^T \phi(s)^{-1}\mathcal{B}_0HdW+\int_0^T \phi(s)^{-1}\bar{g}dW\right\}=0 \end{aligned} \tag{3.53}$$

Introduce an operator $\mathcal{K}: L^2_{\mathcal{F}}(0,T;R_H)\to L_{\mathcal{F}_T}(0,T;R_u)$ as follows

$$\begin{aligned} \mathcal{K}H=\left[\begin{array}{cc} F & -I \end{array}\right]\phi(T)\left\{\int_0^T \phi(s)^{-1}[\mathcal{B}-\bar{A}_0\mathcal{B}_0]Hds\right. \\ \left.+\int_0^T \phi(s)^{-1}\mathcal{B}_0HdW\right\}, \end{aligned} \tag{3.54}$$

then (3.53) can be rewritten as

$$\begin{aligned} 0 &= \left[\begin{array}{cc} F & -I \end{array}\right]\phi(T)\left[\begin{array}{c} x(0) \\ p(0) \end{array}\right]+\mathcal{K}H \\ &\quad +\left[\begin{array}{cc} F & -I \end{array}\right]\phi(T)\left\{\int_0^T \phi(s)^{-1}[\bar{f}-\mathcal{A}_0\bar{g}]ds\right. \\ &\quad \left.+\int_0^T \phi(s)^{-1}\bar{g}dW\right\} \end{aligned} \tag{3.55}$$

The above relation implies that $p(0)$ is affine in $x(0)$.

In addition, let (x_1,p_1,H_1) and (x_2,p_2,H_2) be solutions to (3.42) with initial value x_{10} and x_{20} respectively. It is not difficult to check that $(x_1,p_1,H_1)-(x_2,p_2,H_2)$ is the solution to the homogeneous system (3.45) with initial value $x_{10}-x_{20}$. In addition, there holds

$$p_1-p_2=L(x_1-x_2) \tag{3.56}$$

Hence, L is independent of f_1,f_2,g_1,g_2. In order to characterize L, we only need to make Ito differential for $p=Lx$ and make use of (3.44)-(3.45) and then compare the coefficients.

Theorem 3.1 *The LQR problem for the stochastic system (3.31) is solvable if and only if Riccati equation*

$$\dot{P}+A'P+PA+A_0'PA_0-K_2R_2K_2'=0 \tag{3.57}$$

with

$$R_2=R+B_0'PB_0\geq 0 \tag{3.58}$$
$$K_2=(A_0'PB_0+PB)R_2^+ \tag{3.59}$$
$$PB+A_0'PB_0=R_2R_2^+(A_0'PB_0+PB) \tag{3.60}$$

has a solution. Moreover, the optimal controller is characterized as

$$\begin{aligned} u(t) &= -(R+B_0'PB_0)^+(B'P+B_0'PA_0)x(t) \\ &\quad +S-(R+B_0'PB_0)(R+B_0'PB_0)^+S \end{aligned} \tag{3.61}$$

where S is any vector in R_u.

Proof Nessecity: As the LQR for stochastic systems (3.31) is solvable, with the priori discussion, the trajectories and the optimal controller should admit (3.38)-(3.39) and have a solution. substituting

$$p=Px \tag{3.62}$$

into the co-state equation in (3.38) will lead to

$$\begin{aligned} &-[Qx(t)+A'Px(t)+A_0'H]dt+HdW \\ &= \dot{P}xdt+P([Ax+Bu]dt+[A_0x+B_0u]dW) \end{aligned} \tag{3.63}$$

which means

$$H=P(A_0x(t)+B_0u(t)) \tag{3.64}$$
$$Qx(t)+A'Px(t)+A_0'H=-\dot{P}x-P(Ax+Bu) \tag{3.65}$$

Associating (3.64) with (3.39) , the optimal controller should admit

$$(R+B_0'PB_0)u=-(B'P+B_0'PA_0)x \tag{3.66}$$

Hence, the optimal controller is characterized as (3.39). Plugging (3.61) and (3.64) into (3.65) yields

$$\begin{aligned} &0=[Qx+A'P+A'P+A_0'PA_0 \\ &-(PB+A_0'PB_0)(R+B_0'PB_0)^+(B'P+B_0'PA_0)]x \\ &+(PB+A_0'PB_0)(S-(R+B_0'PB_0)(R+B_0'PB_0)^+S) \end{aligned} \tag{3.67}$$

Since x and S are arbitrary, (3.57) and (3.60) hold, respectively. To prove (3.58), introduce the equality

$$J=J+E[\int_0^T d(x'Px)-x'Px|_0^T]\geq 0 \tag{3.68}$$

with P satisfying (3.57) and (3.60). Direct calculation shows that

$$J=\int_0^T (u-u^*)(R+B_0'PB_0)(u-u^*)dt+x_0'P(0)x_0 \tag{3.69}$$

with

$$\begin{aligned} u^* &= (R+B_0'PB_0)^+(B'P+B_0'PA_0)x \\ &\quad +(S-(R+B_0'PB_0)(R+B_0'PB_0)^+S) \end{aligned}$$

From (3.69), the solvability of $min_u J$ implies the condition (3.58) holds.

Sufficiency: Introduce

$$V(x)=x'Px, P(T)=P_T, \tag{3.70}$$

and

$$\begin{aligned} M &= (PB+A_0'PB_0) \\ &\quad \times[S-(R+B_0'PB_0)^+(R+B_0'PB_0)S] \end{aligned} \tag{3.71}$$

then

$$\begin{aligned} J &= J+E[\int_0^T dV(x)-V(x)|_0^T] \\ &= \int_0^T [(u-v^*)(R+B_0'PB_0)(u-v^*) \\ &\quad +x'\mathcal{L}(P)x+F(M)]dt+x_0'P(0)x_0 \end{aligned} \tag{3.72}$$

where

$$\begin{aligned} v^* &= (R+B_0'PB_0)^+(B'P+B_0'PA_0)x \\ &\quad +(S-(R+B_0'PB_0)(R+B_0'PB_0)^+S) \\ \mathcal{L}(P) &= \dot{P}+A'P+PA+A_0'PA_0 \\ &\quad -(PB+A_0'PB_0)(R+B_0'PB_0)^+(B'P+B_0'PA_0), \end{aligned} \tag{3.73}$$

and $F(M)$ is linear over M, the concrete expression is not important. In view of (3.72), it is clear $\min J_u$ is solvable if $\mathcal{L}(P)=0$, $R+B_0'PB_0\geq 0$ and $A_0'PB_0+PB=(R+B_0'PB_0)(R+B_0'PB_0)^+(A_0'PB_0+PB)=0$. Now the sufficiency is straightforward.

4 Conclusion

The paper considers the complete solution to the ILQR problem for stochastic systems. The LQR problem is solved by virtue of two-point boundary value problems for stochastic differential systems so as to on one hand make the derivation with respect to necessary condition more accessible for readers, on the other hand show the difference between stochastic problems and deterministic ones clearly. It should be stressed that the idea can be used for the most general LQR, where the weighting matrices in cost function may be indefinite and even negative definite. It means that the H_∞ control and game problems with quadratic cost function for stochastic linear systems can be solved similarly. In addition, in the case that the optimal controller is not unique, all the optimal controller is parameterized explicitly.

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1. Analytic solution to indefinite linear quadratic regulator for stochastic systems

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Abstract: This paper aims to deal with the indefinite linear quadratic regulator(ILQR) for stochastic systems. It provides both the analytic solution to the ILQR and the sufficient and necessary condition under which the ILQR is solvable. Different from the existed literature, we obtain the results in a novel way. In order to obtain a tighter necessary condition, we investigate the solution structure of a two-point boundary value problem for a stochastic differential equations directly but involve no complicated probabilistic derivation. Consider that there are close relationships between the linear game problem and H#control and ILQR, the idea in the paper can also be extended to solve these problems. © 2017 Technical Committee on Control Theory, CAA.
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